A note on σ-reversibility and σ-symmetry of skew power series rings

L'moufadal Ben Yakoub and Mohamed Louzari

Department of mathematics
Abdelmalek Essaadi University
B.P. 2121 Tetouan, Morocco
benyakoub@hotmail.com, mlouzari@yahoo.com

Abstract

Let R be a ring and σ an endomorphism of R. In this note, we study the transfert of the symmetry (σ-symmetry) and reversibility (σ-reversibility) from R to its skew power series ring $R[[x;\sigma]]$. Moreover, we study on the relationship between the Baerness, quasi-Baerness and p.p.-property of a ring R and these of the skew power series ring $R[[x;\sigma]]$ in case R is right σ-reversible. As a consequence we obtain a generalization of [10].

Mathematics Subject Classification: 16S36; 16W20; 16U80

Keywords: Armendariz rings; Baer rings; p.p.-rings; quasi-Baer rings; skew power series rings; reversible rings; symmetric rings

1 Introduction

Throughout this paper R denotes an associative ring with identity and σ denotes a nonzero non identity endomorphism of a given ring.

Recall that a ring is *reduced* if it has no nonzero nilpotent elements. Lambek [16], called a ring R *symmetric* if $abc = 0$ implies $acb = 0$ for $a, b, c \in R$. Every reduced ring is symmetric ([19, Lemma 1.1]) but the converse does not hold by [1, Example II.5]. Cohen [8], called a ring R *reversible* if $ab = 0$ implies $ba = 0$ for $a, b \in R$. It is obvious that commutative rings are symmetric and symmetric rings are reversible, but the converse does not hold by [1, Examples I.5 and II.5] and [17, Examples 5 and 7]. From [3], a ring R is called *right* (*left*) *σ-reversible* if whenever $ab =$ for $a, b \in R$, $b\sigma(a) = 0$ ($\sigma(b)a = 0$). R is called

σ-reversible if it is both right and left σ-reversible. Also, by [15], a ring R is called *right* (*left*) *σ-symmetric* if whenever $abc = 0$ for $a, b, c \in R$, $ac\sigma(b) = 0$ ($\sigma(b)ac = 0$). R is called σ-symmetric if it is both right and left σ-symmetric. Clearly right σ-symmetric rings are right σ-reversible.

Rege and Chhawchharia [18], called a ring R an *Armendariz* if whenever polynomials $f = \sum_{i=0}^{n} a_i x^i$, $g = \sum_{j=0}^{m} b_j x^j \in R[x]$ satisfy $fg = 0$, then $a_i b_j = 0$ for each i, j. The Armendariz property of a ring was extended to one of skew polynomial ring in [11]. For an endomorphism σ of a ring R, a *skew polynomial* ring (also called an *Ore extension of endomorphism type*) $R[x;\sigma]$ of R is the ring obtained by giving the polynomial ring over R with the new multiplication $xr = \sigma(r)x$ for all $r \in R$. Also, a *skew power series ring* $R[[x;\sigma]]$ is the ring consisting of all power series of the form $\sum_{i=0}^{\infty} a_i x^i$ ($a_i \in R$), which are multiplied using the distributive law and the Ore commutation rule $xa = \sigma(a)x$, for all $a \in R$. According to Hong et al. [11], a ring R is called σ-skew Armendariz if whenever polynomials $f = \sum_{i=0}^{n} a_i x^i$ and $g = \sum_{j=0}^{m} b_j x^j$ $\in R[x;\sigma]$ satisfy $fg = 0$ then $a_i \sigma^i(b_j) = 0$ for each i, j. Baser et al. [4], introduced the concept of σ-(sps) Armendariz rings. A ring R is called σ-(sps) *Armendariz* if whenever $pq = 0$ for $p = \sum_{i=0}^{\infty} a_i x^i$, $q = \sum_{j=0}^{\infty} b_j x^j \in R[[x;\sigma]]$, then $a_i b_j = 0$ for all i and j. According to Krempa [14], an endomorphism σ of a ring R is called *rigid* if $a\sigma(a) = 0$ implies $a = 0$ for all $a \in R$. We call a ring R *σ-rigid* if there exists a rigid endomorphism σ of R. Note that any rigid endomorphism of a ring R is a monomorphism and σ-rigid rings are reduced by Hong et al. [10]. Also, by [15, Theorem 2.8(1)], a ring R is σ-rigid if and only if R is semiprime right σ-symmetric and σ is a monomorphisme, so right σ-symmetric (σ-reversible) rings are a generalization of σ-rigid rings.

In this note, we introduce the notion of *σ-skew* (*sps*) *Armendariz* rings which is a generalization of σ-(sps) Armendariz rings, and we study the transfert of the symmetry (σ-symmetry) and reversibility (σ-reversibility) from R to its skew power series ring $R[[x;\sigma]]$. Also we show that R is σ-(sps) Armendariz if and only if R is σ-skew (sps) Armendariz and $a\sigma(b) = 0$ implies $ab = 0$ for $a, b \in R$. Moreover, we study on the relationship between the Baerness, quasi-Baerness and p.p.-property of a ring R and these of the skew power series ring $R[[x;\sigma]]$ in case R is right σ-reversible. As a consequence we obtain a generalization of [10].

2 σ-Reversibility and σ-Symmetry of Skew Power Series Rings

We introduce the next definition.

Definition 2.1. *Let R be a ring and σ an endomorphism of R. A ring R*

is called σ-skew (sps) Armendariz if whenever $pq = 0$ for $p = \sum_{i=0}^{\infty} a_i x^i$, $q = \sum_{j=0}^{\infty} b_j x^j \in R[[x;\sigma]]$, then $a_i\sigma^i(b_j) = 0$ for all i and j.

Every subring S with $\sigma(S) \subseteq S$ of an σ-skew (sps) Armendariz ring is a σ-skew (sps) Armendariz ring. In the next, we give an example of a ring R which is σ-skew (sps) Armendariz but not σ-(sps) Armendariz.

Example 2.2. *Let R be the polynomial ring $\mathbb{Z}_2[x]$ over $\mathbb{Z}_2$, and let the endomorphism $\sigma\colon R \to R$ be defined by $\sigma(f(x)) = f(0)$ for $f(x) \in \mathbb{Z}_2[x]$.*
(i) R is not σ-(sps) Armendariz because σ is not a monomorphism.
(ii) R is an σ-skew (sps) Armendariz ring (as in [11, Example 5]). Consider $R[[y;\sigma]] = \mathbb{Z}_2[x][[y;\sigma]]$. Let $p = \sum_{i=0}^{\infty} f_i y^i$ and $q = \sum_{j=0}^{\infty} g_j y^j \in R[[y;\sigma]]$. We have $pq = \sum_{\ell\geq 0}\sum_{\ell=i+j} f_i\sigma^i(g_j)y^\ell = 0$. If $pq = 0$ then $\sum_{\ell=i+j} f_i\sigma^i(g_j)y^\ell = 0$, for each $\ell \geq 0$. Suppose that there is $f_s \neq 0$ for some $s \geq 0$ and $f_0 = f_1 = \cdots = f_{s-1} = 0$, then $\sum_{i+j=s} f_i\sigma^i(g_j)y^{i+j} = 0 \Rightarrow f_s\sigma^s(g_0) = 0$, since R is a domain then $g_0(0) = 0$. Also $\sum_{i+j=s+1} f_i\sigma^i(g_j)y^{i+j} = 0 \Rightarrow f_s\sigma^s(g_1) + f_{s+1}\sigma^{s+1}(g_0) = 0$, since $g_0(0) = 0$ then $f_s\sigma^s(g_1) = 0$ and so $g_1(0) = 0$ by the same method as above. Continuing this process, we have $g_j(0) = 0$ for all $j \geq 0$. Thus $f_i\sigma^i(g_j) = 0$ for all i, j.

We say that R satisfies the condition $(\mathcal{C}_\sigma)$, if whenever $a\sigma(b) = 0$ for $a, b \in R$, then $ab = 0$. By [4, Theorem 3.3(3iii)], if R is σ-(sps) Armendariz then it satisfies $(\mathcal{C}_\sigma)$ (so σ is a monomorphism). If R is an σ-skew (sps) Armendariz ring satisfying the condition $(\mathcal{C}_\sigma)$ then R is σ-(sps) Armendariz.

Theorem 2.3. *A ring R is σ-(sps) Armendariz ring if and only if it is σ-skew (sps) Armendariz and satisfies the condition $(\mathcal{C}_\sigma)$.*

Proof. ($\Leftarrow$). It is clear. ($\Rightarrow$). If R is σ-(*sps*) Armendariz then it satisfies the condition $(\mathcal{C}_\sigma)$. It suffices to show that if R is σ-(*sps*) Armendariz then it is σ-skew (sps) Armendariz. The proof is similar as of [12, Theorem 1.8]. Let $p = \sum_{i=0}^{\infty} a_i x^i$ and $q = \sum_{j=0}^{\infty} b_j x^j \in R[[x;\sigma]]$ with $pq = 0$. Note that $a_j b_j = 0$ for all i and j. We claim that $a_i\sigma^i(b_j) = 0$ for all i and j. We have $(a_0 + a_1x + \cdots)(b_0 + b_1x + \cdots) = 0$, then $a_0(b_0 + b_1x + \cdots) + (a_1x + a_2x^2 \cdots)(b_0 + b_1x + \cdots) = 0$. Since $a_0 b_j = 0$ for all j, we get

$$0 = (a_1x + a_2x^2 + \cdots)(b_0 + b_1x + \cdots)$$

$$0 = (a_1 + a_2x + \cdots)x(b_0 + b_1x + \cdots)$$
$$0 = (a_1 + a_2x + \cdots)(\sigma(b_0)x + \sigma(b_1)x^2 + \cdots).$$

Put $p_1 = a_1 + a_2x + \cdots$ and $q_1 = \sigma(b_0)x + \sigma(b_1)x^2 + \cdots$. Since $p_1q_1 = 0$ then $a_i\sigma(b_j) = 0$ for all $i \geq 1$ and $j \geq 0$. We have, also

$$0 = a_1(\sigma(b_0)x + \sigma(b_1)x^2 + \cdots) + (a_2x + a_3x^2 + \cdots)(\sigma(b_0)x + \sigma(b_1)x^2 + \cdots).$$

Since $a_1\sigma(b_j) = 0$ for all j, then

$$0 = (a_2x + a_3x^2 + \cdots)(\sigma(b_0)x + \sigma(b_1)x^2 + \cdots)$$
$$0 = (a_2 + a_3x + \cdots)(\sigma^2(b_0)x^2 + \sigma^2(b_1)x^3 + \cdots).$$

Put $p_2 = a_2 + a_3x + a_4x^2 + \cdots$ and $q_2 = \sigma^2(b_0)x^2 + \sigma^2(b_1)x^3 + \cdots$, and then $p_2q_2 = 0$ implies $a_i\sigma^2(b_j) = 0$ for all $i \geq 2$ and $j \geq 0$. Continuing this process, we can show that $a_i\sigma^i(b_j) = 0$ for all $i \geq 0$ and $j \geq 0$. Thus R is σ-skew (*sps*) Armendariz. □

Lemma 2.4. *Let R be an σ-(sps) Armendariz ring. Then for $f = \sum_{i=0}^{\infty} a_ix^i$, $g = \sum_{j=0}^{\infty} b_jx^j$ and $h = \sum_{k=0}^{\infty} c_kx^k \in R[[x;\sigma]]$, if $fgh = 0$ then $a_ib_jc_k = 0$ for all i, j, k.*

Proof. Note that, if $fg = 0$ then $a_ig = 0$ for all i. Suppose that $fgh = 0$ then $a_i(gh) = 0$ for all i, and so $(a_ig)h = 0$ for all i. Therefore $a_ib_jc_k = 0$ for all i, j, k. □

Proposition 2.5. *Let R be an σ-(sps) Armendariz ring. Then*
(1) *R is reversible if and only if $R[[x;\sigma]]$ is reversible.*
(2) *R is symmetric if and only if $R[[x;\sigma]]$ is symmetric.*

Proof. If $R[[x;\sigma]]$ is symmetric (reversible) then R is symmetric (reversible). Conversely, (1). Let $f = \sum_{i=0}^{\infty} a_ix^i$ and $g = \sum_{j=0}^{\infty} b_jx^j \in R[[x;\sigma]]$, if $fg = 0$ then $a_ib_j = 0$ for all i and j. By [4, Theorem 3.3 (3ii)], we have $\sigma^j(a_i)b_j = 0$ for all i and j. Since R is reversible, we obtain $b_j\sigma^j(a_i) = 0$ for all i and j. Thus $gf = \sum_{\ell=0}^{\infty}\sum_{\ell=i+j} b_j\sigma^j(a_i)x^\ell = 0$. (2). We will use freely [4, Theorem 3.3 (3ii)], reversibility and symmetry of R. Let $f = \sum_{i=0}^{\infty} a_ix^i$, $g = \sum_{j=0}^{\infty} b_jx^j$ and $h = \sum_{k=0}^{\infty} c_kx^k \in R[[x;\sigma]]$, if $fgh = 0$ then $a_ib_jc_k = 0$ for all i, j and k, by Lemma 2.4. Then for all i, j, k we have $b_jc_ka_i = 0 \Rightarrow \sigma^k(b_j)c_ka_i = 0 \Rightarrow a_i\sigma^k(b_j)c_k = 0 \Rightarrow a_ic_k\sigma^k(b_j) = 0 \Rightarrow c_k\sigma^k(b_j)a_i = 0 \Rightarrow \sigma^i[c_k\sigma^k(b_j)]a_i = 0 \Rightarrow a_i\sigma^i[c_k\sigma^k(b_j)] = 0$. Thus $fhg = 0$. □

The next Lemma gives a relationship between σ-reversibility (σ-symmetry) and reversibility (symmetry).

Lemma 2.6 ([5, Lemma 3.1])**.** *Let R be a ring and σ an endomorphism of R. If R satisfies the condition $(\mathcal{C}_\sigma)$. Then*
(1) *R is reversible if and only if R is σ-reversible;*
(2) *R is symmetric if and only if R is σ-symmetric.*

Theorem 2.7. *Let R be an σ-(sps) Armendariz ring. The following statements are equivalent:*
(1) *R is reversible (symmetric);*
(2) *R is σ-reversible (σ-symmetric);*
(3) *R is right σ-reversible (right σ-symmetric);*
(4) *$R[[x;\sigma]]$ is reversible (symmetric).*

Proof. We prove the reversible case (the same for the symmetric case).
(1) $\Leftrightarrow$ (4). By Proposition 2.5.
(1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3). Immediately from Lemma 2.6.
(3) $\Rightarrow$ (1). Let $a, b \in R$, if $ab = 0$ then $b\sigma(a) = 0$ (right σ-reversibility), so $ba = 0$ (condition $(\mathcal{C}_\sigma)$). $\square$

3 Related Topics

In this section we turn our attention to the relationship between the Baerness, quasi-Baerness and p.p.-property of a ring R and these of the skew power series ring $R[[x;\sigma]]$ in case R is right σ-reversible. For a nonempty subset X of R, we write $r_R(X) = \{c \in R | dc = 0 \text{ for any } d \in X\}$ which is called the right annihilator of X in R.

Lemma 3.1. *If R is a right σ-reversible ring with $\sigma(1) = 1$. Then*
(1) *$\sigma(e) = e$ for all idempotent $e \in R$;*
(2) *R is abelian.*

Proof. (1) Let e an idempotent of R. We have $e(1-e) = (1-e)e = 0$ then $(1-e)\sigma(e) = e\sigma((1-e)) = 0$, so $\sigma(e) - e\sigma(e) = e - e\sigma(e) = 0$, therefore $\sigma(e) = e$. (2) Let $r \in R$ and e an idempotent of R. We have $e(1-e) = 0$ then $e(1-e)r = 0$, since R is right σ-reversible then $(1-e)r\sigma(e) = 0 = (1-e)re = 0$, so $re = ere$. Since $(1-e)e = 0$, we have also $er = ere$. Then R is abelian. $\square$

Lemma 3.2. *Let R be a right σ-reversible ring with $\sigma(1) = 1$, then the set of all idempotents in $R[[x;\sigma]]$ coincides with the set of all idempotents of R. In this case $R[[x;\sigma]]$ is abelian.*

Proof. We adapt the proof of [3, Theorem 2.13(iii)] for $R[[x;\sigma]]$. Let $f^2 = f \in R[[x;\sigma]]$, where $f = f_0 + f_1x + f_2x^2 + \cdots$. Then

$$\sum_{\ell=0}^{\infty} \sum_{\ell=i+j} f_i\sigma^i(f_j)x^\ell = \sum_{\ell=0}^{\infty} f_\ell x^\ell.$$

For $\ell = 0$, we have $f_0^2 = f_0$. For $\ell = 1$, we have $f_0f_1 + f_1\sigma(f_0) = f_1$, but f_0 is central and $\sigma(f_0) = f_0$, so $f_0f_1 + f_1f_0 = f_1$, a multiplication by $(1-f_0)$ on the left hand gives $f_1 = f_0f_1$, and so $f_1 = 0$. For $\ell = 2$, we have $f_0f_2 + f_1\sigma(f_1) + f_2\sigma^2(f_0) = f_2$, so $f_0f_2 + f_2f_0 = f_2$ (because $f_1 = 0$ and $\sigma^2(f_0) = f_0$), a multiplication by $(1-f_0)$ on the left hand gives $f_0f_2 = f_2 = 0$. Continuing this procedure yields $f_i = 0$ for all $i \geq 1$. Consequently, $f = f_0 = f_0^2 \in R$. Since R is abelian then $R[[x;\sigma]]$ is abelian. $\square$

Kaplansky [13], introduced the concept of *Baer rings* as rings in which the right (left) annihilator of every nonempty subset is generated by an idempotent. According to Clark [7], a ring R is called *quasi-Baer* if the right annihilator of each right ideal of R is generated (as a right ideal) by an idempotent. It is well-known that these two concepts are left-right symmetric. A ring R is called a *right* (*left*) *p.p.-ring* if the right (left) annihilator of an element of R is generated by an idempotent. R is called a *p.p.-ring* if it is both a right and left p.p.-ring.

Theorem 3.3. *Let R be a right σ-reversible ring with $\sigma(1) = 1$. Then*
(1) *R is a Baer ring if and only if $R[[x;\sigma]]$ is a Baer ring;*
(2) *R is a quasi-Baer ring if and only if $R[[x;\sigma]]$ is a quasi-Baer ring.*

Proof. ($\Rightarrow$). Suppose that R is Baer. Let A be a nonempty subset of $R[[x;\sigma]]$ and A^* be the set of all coefficients of elements of A. Then A^* is a nonempty subset of R and so $r_R(A^*) = eR$ for some idempotent element $e \in R$. Since $e \in r_{R[[x;\sigma]]}(A)$ by Lemma 3.1. We have $eR[[x;\sigma]] \subseteq r_{R[[x;\sigma]]}(A)$. Now, let $0 \neq q = b_0 + b_1x + b_2x^2 + \cdots \in r_{R[[x;\sigma]]}(A)$. Then $Aq = 0$ and hence $pq = 0$ for any $p \in A$. Let $p = a_0 + a_1x + a_2x^2 + \cdots$, then

$$pq = \sum_{\ell\geq 0}\sum_{\ell=i+j} a_i\sigma^i(b_j)x^\ell = 0.$$

- $\ell = 0$ implies $a_0b_0 = 0$ then $b_0 \in r_R(A^*) = eR$.
- $\ell = 1$ implies $a_0b_1 + a_1\sigma(b_0) = 0$, since $b_0 = eb_0$ and $\sigma(e) = e$ then $a_0b_1 + a_1e\sigma(b_0) = 0$, but $a_1e = 0$ so $a_0b_1 = 0$ and hence $b_1 \in r_R(A^*)$.
- $\ell = 2$ implies $a_0b_2 + a_1\sigma(b_1) + a_2\sigma^2(b_0) = 0$, then $a_0b_2 + a_1e\sigma(b_1) + a_2e\sigma^2(b_0) = 0$, but $a_1e\sigma(b_1) = a_2e\sigma^2(b_0) = 0$, hence $a_0b_2 = 0$. Then $b_2 \in r_R(A^*)$.

Continuing this procedure yields $b_0, b_1, b_2, b_3, \cdots \in r_R(A^*)$. So, we can write $q = eb_0 + eb_1x + eb_2x^2 + \cdots \in eR[[x;\sigma]]$. Therefore $eR[[x;\sigma]] = r_{R[[x;\sigma]]}(A)$. Consequently, $R[[x;\sigma]]$ is a Baer ring.

Conversely, Suppose that $R[[x;\sigma]]$ is Baer. Let B be a nonempty subset of R. Then $r_{R[[x;\sigma]]}(B) = eR[[x;\sigma]]$ for some idempotent $e \in R$ by Lemma 3.2. Thus $r_R(B) = r_{R[[x;\sigma]]}(B) \cap R = eR[[x;\sigma]] \cap R = eR$. Therefore R is Baer.

The proof for the case of the quasi-Baer property follows in a similar fashion; In fact, for any right ideal A of $R[[x;\sigma]]$, take A^* as the right ideal generated by all coefficients of elements of A. $\square$

From [10, Example 20], $R = M_2(\mathbb{Z})$ is a Baer ring and $R[[x]]$ is not Baer. Clearly R is not reversible. So that, the "right σ-reversibility" condition in Theorem 3.3(1) is not superfluous.

According to Annin [2], a ring R is *σ-compatible* if for each $a, b \in R$, $a\sigma(b) = 0$ if and only if $ab = 0$. Hashemi and Moussavi [9, Corollary 2.14] have proved Theorem 3.3(2), when R is σ-compatible. Consider R and σ as in Example 2.2. Since R is a domain then it is right σ-reversible (with $\sigma(1) = 1$). Also R is not σ-compatible (so R does not satisfy the condition $(\mathcal{C}_\sigma)$), because σ is not a monomorphism. Therefore Theorem 3.3(2) is not a consequence of [9, Corollary 2.14]. On other hand, if R is reversible then σ-compatibility implies right σ-reversibility. But, if R is not reversible, we can easily see that this implication does not hold.

Theorem 3.4. *Let R be a right σ-reversible ring with $\sigma(1) = 1$. If $R[[x; \sigma]]$ is a p.p.-ring then R is a p.p.-ring.*

Proof. Suppose that $R[[x; \sigma]]$ is a right p.p.-ring. Let $a \in R$, then there exists an idempotent $e \in R$ such that $r_{R[[x;\sigma]]}(a) = eR[[x; \sigma]]$ by Lemma 3.2. Hence $r_R(a) = eR$, and therefore R is a right p.p.-ring. □

Also, in Example 2.2, R is not σ-(sps) Armendariz. So Theorem 3.3 and Theorem 3.4 are not consequences of [4, Theorem 3.2].

Since σ-rigid rings are right σ-reversible [15, Theorem 2.8 (1)], we have the following Corollaries.

Corollary 3.5 ([10, Theorem 21]). *Let R be an σ-rigid ring. Then R is a Baer ring if and only if $R[[x; \sigma]]$ is a Baer ring.*

Corollary 3.6 ([10, Corollary 22]). *Let R be an σ-rigid ring. Then R is a quasi-Baer ring if and only if $R[[x; \sigma]]$ is a quasi-Baer ring.*

ACKNOWLEDGEMENTS. The second author wishes to thank Professor Amin Kaidi of University of Almería for his generous hospitality. This work was supported by the project PCI Moroccan-Spanish A/011421/07.

References

[1] D.D. Anderson and V. Camillo, Semigroups and rings whose zero products commute, *Comm. Algebra,* **27(6)** (1999), 2847-2852.

[2] S. Annin, Associated primes over skew polynomials rings, *Comm. Algebra,* bf 30 (2002), 2511-2528.

[3] M. Baser, C.Y. Hong and T.K. Kwak, On extended reversible rings, *Algebra Colloq.,* **16(1)** (2009), 37-48.

[4] M. Baser, A. Harmanci and T.K. Kwak, Generalized semicommutative rings and their extensions, *Bull. Korean Math. Soc.,* **45(2)** (2008), 285-297.

[5] L. Ben Yakoub and M. Louzari, Ore extensions of extended symmetric and reversible rings, *Inter. J. of Algebra,* (to appear).

[6] G.F. Birkenmeier, J.Y. Kim, J.K. Park, Principally quasi-Baer rings, *Comm. Algebra,* **29(2)** (2001), 639-660.

[7] W.E. Clark, Twisted matrix units semigroup algebras, *Duke Math.Soc.,* **35** (1967), 417-424.

[8] P.M. Cohen, Reversible rings, *Bull. London Math. Soc.,* **31(6)** (1999), 641-648.

[9] E. Hashemi and A. Moussavi, Polynomial extensions of quasi-Baer rings, *Acta. Math. Hungar.,* **107 (3)** (2005), 207-224.

[10] C.Y. Hong, N.K. Kim and T.K. Kwak, Ore extensions of Baer and p.p.-rings, *J. Pure and Appl. Algebra,* **151(3)** (2000), 215-226.

[11] C.Y. Hong, N.K. Kim and T.K. Kwak, On Skew Armendariz Rings, *Comm. Algebra,* **31(1)** (2003), 103-122.

[12] C.Y. Hong, T.K. Kwak and S.T. Rezvi, Extensions of generalized Armendariz rings, *Algebra Colloq.,* **13(2)** (2006), 253-266.

[13] I. Kaplansky, *Rings of operators,* Math. Lecture Notes series, Benjamin, New York, 1965.

[14] J. Krempa, Some examples of reduced rings, *Algebra Colloq.,* **3(4)** (1996), 289-300.

[15] T.K. Kwak, Extensions of extended symmetric rings, *Bull. Korean Math.Soc.,* **44** (2007), 777-788.

[16] J. Lambek, On the reprentation of modules by sheaves of factor modules, *Canad. Math. Bull.,* **14** (1971), 359-368.

[17] G. Marks, Reversible and symmetric rings, *J. Pure Appl. Algebra,* **174(3)** (2002), 311-318.

[18] M.B. Rege and S. Chhawchharia, Armendariz rings, *Proc. Japan. Acad. Ser. A Math. Sci.,* **73** (1997), 14-17.

[19] J.Y. Shin, Prime ideals and sheaf representation of a pseudo symmetric ring, *Trans. Amer. Math. Soc.,* **184** (1973), 43-60.